Chapter 4
Kinematics Chapter Review

EQUATIONS:

- $\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$ [This is the formal presentation of a position vector in unit vector coordinates. It identifies the position of an object at a given instant. Example: $-3\boldsymbol{i} + 4\boldsymbol{j} - 5\boldsymbol{k}$, where *-3* is the *x component* of the object's position, *+4* is the *y component* of the object's position, and *-5* is the *z component* of the object's position. Note that the units of a position vector are *meters*.]

- $\boldsymbol{v} = v_x\boldsymbol{i} + v_y\boldsymbol{j} + v_z\boldsymbol{k}$ [This is the formal presentation of a velocity vector given in unit vector notation. It identifies the rate of position change and the direction in which an object is traveling at a given instant. Example: $-3\boldsymbol{i} + 4\boldsymbol{j} - 5\boldsymbol{k}$, where *-3* is the *x component* of the rate of position change with time, *+4* is the *y component* of the rate of position change with time, and *-5* is the *z component* of the rate of position change with time. Note that the units of velocity are *meters per second*.]

- $\boldsymbol{a} = a_x\boldsymbol{i} + a_y\boldsymbol{j} + a_z\boldsymbol{k}$ [This is the formal presentation of an acceleration vector, given in unit vector notation. It identifies the rate at which an object's velocity changes at a given instant. Example: $-3\boldsymbol{i} + 4\boldsymbol{j} - 5\boldsymbol{k}$, where *-3* is the *x component* of the rate of velocity change with time, *+4* is the *y component* of the rate of velocity change with time, and *-5* is the *z component* of the rate of velocity change with time. Note that the units of acceleration are *meters per second per second*.]

- $v_x = \frac{dx}{dt}$ [This is the relationship between the velocity in the *x direction* and the rate of position change in the *x direction*.]

- $v_x dt = dx$ [An extension of the expression just above, this is the relationship between a differential (read this *tiny, tiny*) position change *dx*, the instantaneous velocity in the *x direction* during the change, and the differential time interval *dt* over which the change occurs. Note that this *appears* to follow in an algebraic sense from $v_x = \frac{dx}{dt}$, though mathematicians curl up their toes and spout steam from their ears when physicists take such shortcuts. Note also that from $v_x dt = dx$, it follows that $\int_{t_1}^{t_2} v_x dt = \int_{x_1}^{x_2} dx$. For constant velocity, this becomes $v_x(t_2 - t_1) = (x_2 - x_1)$, or $x_2 = x_1 + v_x \Delta t$, or $v_x \Delta t = \Delta x$, depending upon how you want to present the evaluation.]

- $a_x = \frac{dv_x}{dt}$ [This is the relationship between the instantaneous acceleration in the *x direction* and the rate of velocity change in the *x direction*.]

- $a_x dt = dv_x$ [An extension of the expression just above, this is the relationship between a differential velocity change *dv*, the instantaneous acceleration in the *x* direction during the change, and the differential time interval dt over which the change occurred. It also follows that $\int_{t_1}^{t_2} a_x dt = \int_{v_1}^{v_2} dv_x$ . . .]

- $a_x = \frac{d^2x}{dt^2}$ [This is the relationship between the instantaneous acceleration in the *x direction* and the *rate* of the rate of position change in the *x direction*.]

- $v_{avg} = \frac{v_1 + v_2}{2}$ [FOR A CONSTANT ACCELERATION SITUATION, this relates the *average* velocity over a given time interval Δt to the initial and final velocities associated with that interval. This expression is RARELY USED.]

- $v_{avg} = \frac{x_2 - x_1}{\Delta t}$ ***or*** $x_2 = x_1 + v_{avg}\Delta t$ [FOR A CONSTANT ACCELERATION SITUATION, this relates the *average* velocity over a given time interval Δt to the initial and final positions associated with that interval. This expression is RARELY USED.]

- $a_{avg} = a_{instantaneous} = a$ [FOR A CONSTANT ACCELERATION SITUATION, the instantaneous acceleration and the average acceleration are the same, often denoted simply by a.]

- $a = \frac{v_2 - v_1}{\Delta t}$ or $v_2 = v_1 + a\Delta t$ [FOR A CONSTANT ACCELERATION SITUATION, this relates the instantaneous acceleration and the velocity change $(v_2 - v_1)$ over a given time interval Δt. This expression is USED A LOT.)]

- $x_2 = x_1 + v_1(\Delta t) + \frac{1}{2}a(\Delta t)^2$ or $\Delta x = v_1 t + \frac{1}{2}at^2$ [FOR A CONSTANT ACCELERATION SITUATION, this relates the position change over a time interval Δt, the interval's initial velocity, and the acceleration. This expression is USED A LOT. Also, note the somewhat standard notation wherein the Δt terms are simply presented as t terms. This is sloppy but somewhat conventional. My apologies, but get used to it.]

- $v_2{}^2 = v_1{}^2 + 2a(x_2 - x_1)$ or $v_2{}^2 = v_1{}^2 + 2a\Delta x$ [FOR A CONSTANT ACCELERATION SITUATION, this relates the velocities at the extremes of an interval, the position change over that interval, and the system's acceleration. This expression is USED A LOT.]

COMMENTS, HINTS, and THINGS to be aware of:

- **Kinematics** is essentially idiot physics, not because any idiot can do it (in fact, very intelligent people have trouble with it), but because all that is needed to use it successfully is the ability to identify what is given in a problem, to identify what is asked for, and to pick the kinematic equation that incorporates both what is known and what is required. Yes, there may be problems in which you have to solve *two* kinematic equations simultaneously. But on the whole, your ability or inability to use kinematics intelligently will have absolutely no bearing on how well you will do when we begin to look at more formal, advanced approaches to physics.

- The **direction of a velocity vector** at a given instant is the same as the direction of the body's motion at that instant.

- The *net force* on a body and the body's *acceleration* go hand in hand. That is, if the net force on a body is in the negative direction, the acceleration will be negative. With that in mind, consider a body moving in one dimensional motion. If the net force (and, by extension, acceleration) is in the positive direction while the body's velocity is in the negative direction, what will the body do? It will slow down! And if the net force (and, by extension, acceleration) is in the negative direction and the body's velocity is still in the negative direction, what will the body do? It will speed up! So what does the **sign of the acceleration** tell you? Nothing, unless you additionally know the sign of the velocity. If the two are the same (both positive or both negative), the body will speed up. If the two are different, the body will slow down.

- It is not unusual to see the symbol ***t*** in kinematic equations. This stands for a TIME INTERVAL. It should be written as Δt. However, convention and sloppiness have prevailed and the symbol is usually shortened to t.

- You should be able to take a ***velocity versus time*** and corresponding ***position versus time* graph** and pick off all of the information needed to determine, say, the system's acceleration. You should be able to do this TWO or THREE DIFFERENT WAYS using the kinematic relationships as a guide (that is, using $v_2^2 = v_1^2 + 2a\,\Delta x$, or $x_2 = x_1 + v_1 t + .5at^2$, or whatever). In other words, you need to understand what the variables in the kinematic relationships REALLY MEAN.

- You should be able to make sense out of a ***velocity versus time* graph**. That is, you should know that the area under the graph over a particular time interval equals the distance traveled during that interval, and that the slope of the tangent to the curve, evaluated at a particular instant, equals the instantaneous acceleration at that point in time. Both of these observations are true whether the acceleration is constant or not, but when used in conjunction with a constant acceleration, the kinematic equations follow naturally.

- You should be able to make sense out of a ***position versus time* graph**. That is, you should know that the slope of the tangent to the curve, evaluated at a particular instant, equals the instantaneous velocity at that point in time.

- You should be able to make sense out of an ***acceleration versus time* graph**. That is, you should know that the area under the graph over a particular time interval equals the velocity change Δv during that interval.